

A Brief Tutorial Combinatorial Analysis

What is Combinatorial Analysis?

- ▶ To figure out the house edge of a game requires mathematical analysis.
 - ▶ There are two methods of analysis – combinatorial analysis (CA) and simulation.
 - ▶ The preferred method is always (CA) and then double check using a simulation.
 - ▶ Combinatorial analysis requires running a “cycle”.
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What is a Cycle?

- ▶ There is no such thing as an “infinite” game. Every game consists of finitely many ways it can play out.
 - ▶ The total number of ways the game can play out is called a cycle.
 - ▶ To analyze a game requires accounting for every possible way the game can play out.
 - ▶ This can be a very large number.
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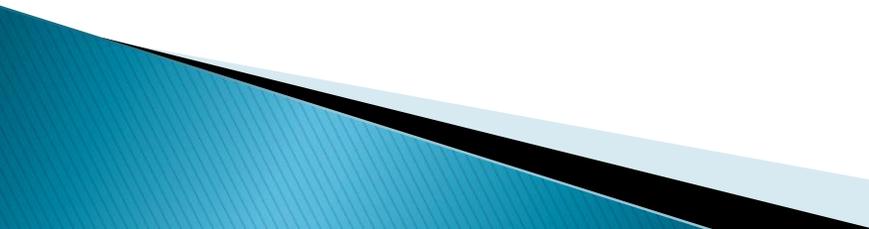
Some Cycles

Game	Cycle Length
Lucky Ladies (2 decks)	27588756
Three Card Poker	407170400
Ultimate Texas Hold'em	27813810024000
Baccarat (8 decks)	4998398275503360

Example: Bust it Blackjack Side Bet

<http://aheat.net/2013/03/11/card-counting-the-bust-it-blackjack-side-bet/>

Example: “Bust It” BJ Side Bet

- ▶ Dealer busts with suited 8, 8, 8 pays 200-to-1.
 - ▶ Dealer busts with colored 8, 8, 8 pays 50-to-1.
 - ▶ Dealer busts with a six pays 15-to-1.
 - ▶ Dealer busts with a seven pays 9-to-1.
 - ▶ Dealer busts with an eight pays 7-to-1.
 - ▶ Dealer busts with a nine pays 5-to-1.
 - ▶ Dealer busts with a ten pays 3-to-1.
 - ▶ Otherwise, the player loses the BI bet.
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Cycle for BI

- ▶ BI is completely analyzed by considering the dealer's two cards and the dealer draw-card.
 - ▶ In two decks, there are 5356 ($104 \text{ choose } 2$) ways the dealer can be dealt two cards.
 - ▶ There are then 102 cards left for the dealer.
 - ▶ A cycle consists of $5356 \times 102 = 546312$ different outcomes.
 - ▶ The cycle length is 546312.
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Bust It Blackjack Side Bet Combinatorics: 2 Decks

Event	N	p	EV	p*EV	VAR
8,8,8 Suited	0	0.00000	200	0.00000	0.00000
8,8,8, Same color	24	0.00004	50	0.00220	0.11016
Bust with Six	2528	0.00463	15	0.06941	1.05170
Bust with Seven	5728	0.01048	9	0.09436	0.86362
Bust with Eight	8944	0.01637	7	0.11460	0.81965
Bust with Nine	12480	0.02284	5	0.11422	0.58852
Bust with Ten	64896	0.11879	3	0.35637	1.12372
Lose	451712	0.82684	-1	-0.82684	0.70643
Total	546312	1.00000		-0.07568	5.26379

Event

- ▶ Each time the game is played, something happens.
 - ▶ The “Events” list all of the different things that can happen each time the game is played.
 - ▶ Every possibility has to be listed, including losing the game.
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N

- ▶ In one cycle, tally the number of ways each single event occurs.
 - ▶ N gives the number of ways the specific event happens in one cycle.
 - ▶ For example, the dealer busting with a nine occurs 12480 times in a cycle.
 - ▶ The sum of all the values under N has to equal the cycle length.
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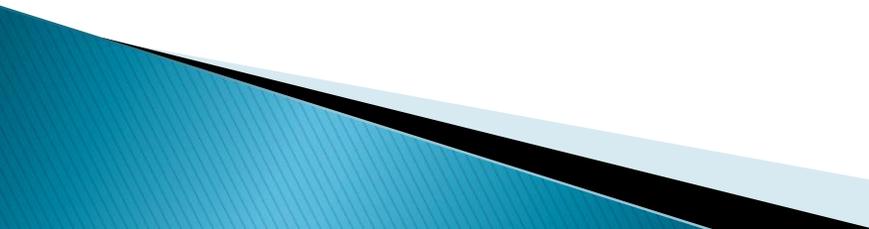
Probability (p)

- ▶ The probability of an event represents the chance that event occurs.
- ▶ We use the notation $p(\text{Event})$ or simply “p” if the even is understood.
- ▶ The definition of $p(\text{Event})$ is:
$$p(\text{Event}) = (\# \text{ of outcomes in event}) / (\text{Cycle length})$$
- ▶ The probability of busting with a nine is:
$$p(\text{bust with 9}) = 12480 / 546312 = 0.02284$$
- ▶ The sum of the probabilities must be 1.

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Expected Value: (EV)

- ▶ For each event that occurs, the EV is simply the amount that event pays.
 - ▶ It is common to use “Pays” rather than “EV” for this column.
 - ▶ For example, the EV of busting with a nine is 5. This means that the player wins 5 units.
 - ▶ Note that the EV for losing BI is -1 .
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$P*EV$

- ▶ The entries in this column are simply the product of the “p” and “EV” values immediately to the left.
- ▶ These values must be computed to determine the overall house edge.
- ▶ The sum of the $p*EV$ values is equal to the house edge, which is the number at the bottom of the “ $p*EV$ ” column.

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House Edge

- ▶ The house edge is the number at the bottom of the “ $p*EV$ ” column.
- ▶ This value is from the player’s perspective, because the “EV” values are from the player’s perspective.
- ▶ A negative value for the house edge means the house has the advantage over the player.
- ▶ The house edge for BI is 7.568%.

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VAR

- ▶ The final column “VAR” represents the variance for each outcome of the game.
- ▶ The value at the bottom of VAR is the sum of the values in the VAR column. It represents the overall variance of the game.
- ▶ VAR is used to compute the standard deviation (SD). $SD = \sqrt{VAR}$.
- ▶ VAR and SD are not part of the CA.

Baccarat Tie Bet, Tie Pays 8-to-1, Combinatorial Analysis					
EVENT	N	p	EV	p*EV	VAR
Tie	475627426473216	0.09516	8	0.76125	6.16396
NonTie	4522770849030140	0.90484	-1	-0.90484	0.81931
Total	4998398275503360	1.00000		-0.14360	6.98326

Baccarat Tie Bet, Tie Pays 9-to-1, Combinatorial Analysis					
EVENT	N	p	EV	p*EV	VAR
Tie	475627426473216	0.09516	9	0.85640	7.79083
NonTie	4522770849030140	0.90484	-1	-0.90484	0.81931
Total	4998398275503360	1.00000		-0.04844	8.61013

Three Card Poker Hole-Card Combinatorics - Full Cycle					
Win	N	Out	p	p*Win	VAR
-2	138570276	-277140552	0.340325	-0.680650	1.409125
-1	47601384	-47601384	0.116908	-0.116908	0.125193
0	229912	0	0.000565	0.000000	0.000001
1	119271408	119271408	0.292928	0.292928	0.272878
2	90682848	181365696	0.222715	0.445429	0.860102
3	8976452	26929356	0.022046	0.066138	0.193833
4	0	0	0.000000	0.000000	0.000000
5	289104	1445520	0.000710	0.003550	0.017504
6	931972	5591832	0.002289	0.013733	0.081447
7	617044	4319308	0.001515	0.010608	0.073520
	407170400	14181184	1.000000	0.034829	3.033603

Comments on CA

- ▶ There are other ways to present CA.
- ▶ Sometimes the “Events” are replaced by the “EV” values if the events are not clear – for example in Three Card Poker or Blackjack.
- ▶ Sometimes “Out” is used in place of $p \cdot EV$. This is common in slot machine analysis.
- ▶ Sometimes a game is too complicated for CA. In that case, simulation is the only option.